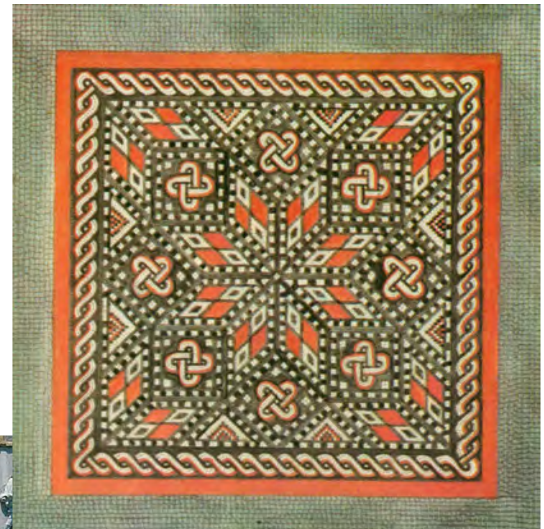
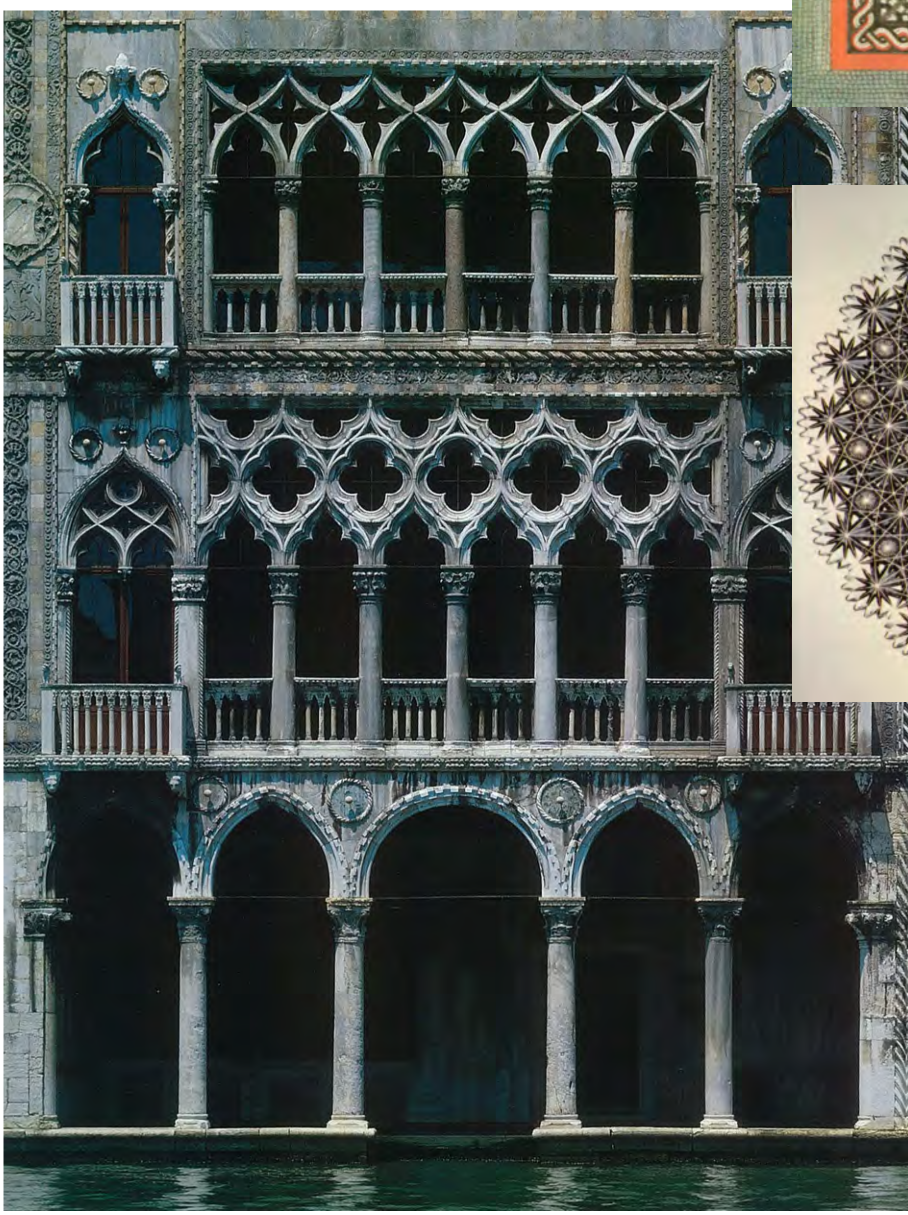
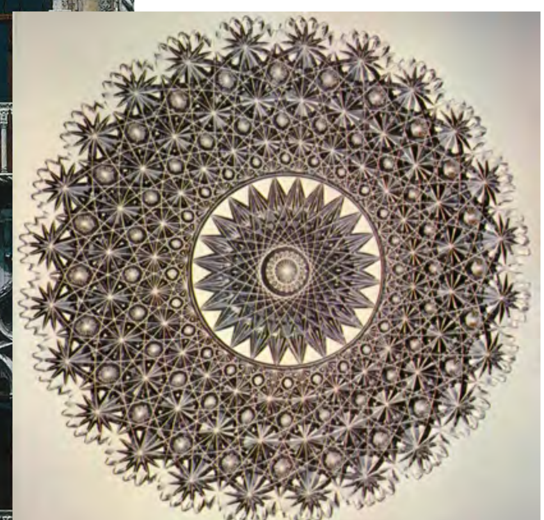
# Section 2
# Basic Constructions

The constructions in this section are the basis of all the remaining constructions in The Library. They are as fundamental to these constructions as the letters of the alphabet are to writing. Their invention lies far back in the most remote periods of human history, and they have been universally employed from the earliest times. These basic constructions should be practiced repeatedly until they are thoroughly mastered and fully memorized.





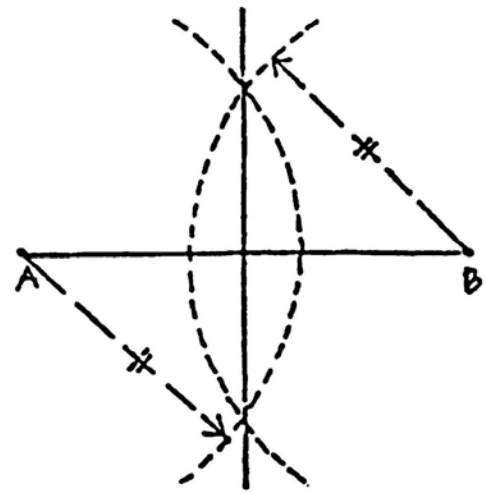


## Overview

Geometry is derived from the term "earth-measure". It is a practical method of accurately measuring and manipulating the elements of the physical world for human use. The origins of geometry predate recorded history.

Examples of pre-historic geometry can be found worldwide. They can be seen in structures such as Stonehenge in England, Great Zimbabwe in Zimbabwe and Ponape in the Caroline Islands of the Pacific, in the frescoes of Catal Huyuk in Turkey, the fabrics of Paracas in Peru, the pottery of the Anazasi in the Southwestern U.S, and in the jewelry of the Scythians in the Ukraine. It is obvious that in all of these examples and countless others, the creators had to "lay out" their design accurately in the real world.

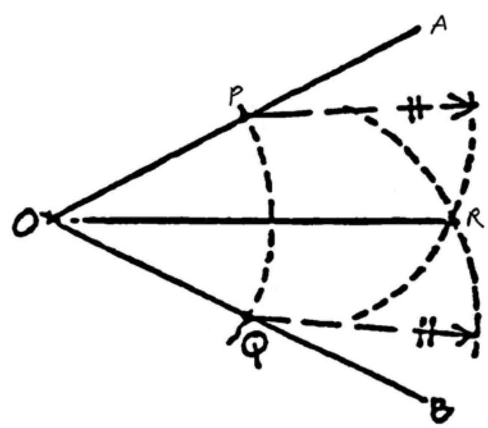
The basic constructions of geometry are therefore the product of the ongoing intellectual development of humanity. With the emergence of literate societies in Egypt, The Near East, India and China, these methods were codified, written down step-by-step in carefully guarded texts. These texts were handed down generation after generation, preserving the hard won discoveries for future users. They became and continue to be the foundation of our ability to order and restructure the physical world. Although to our more sophisticated consciousness, they appear self-evidently simple, they are in fact profoundly complex and are evidence of the capacity of the human mind to develop abstract thought.




**2.1.1 To bisect a line.**

1. Given line AB, set compass at point A and open the compass a distance approximately wider than half length of line AB.
2. Draw arc around A.
3. Keeping compass at same setting, draw arc around point B.
4. Where arcs intersect, join with straight line.

This line will bisect line AB (and be perpendicular to it).

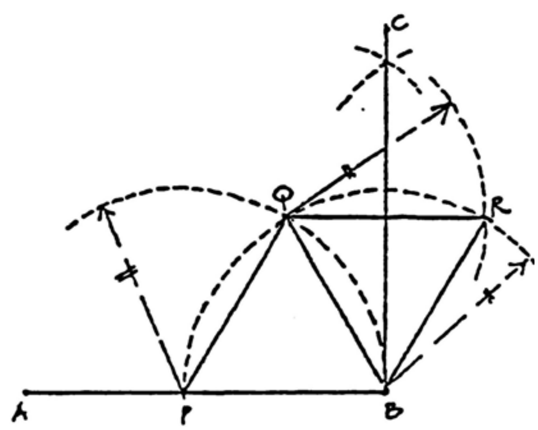



**2.1.2 To bisect a given angle.**

1. Given angle AOB, set compass to any radius, in this case OP, and with O as centre, swing arc OP to point Q on line OB.
2. Keeping compass at same setting, use point P as centre. Swing arc.
3. Keeping compass at same setting, use point Q as centre. Swing arc.
4. The arcs about point P and Q respectively will intersect at point R.
5. Connect point O to point R, the line OR will bisect angle AOB.

**2.1.3 To draw a perpendicular to a straight line from a point at, or near, one of its ends.**

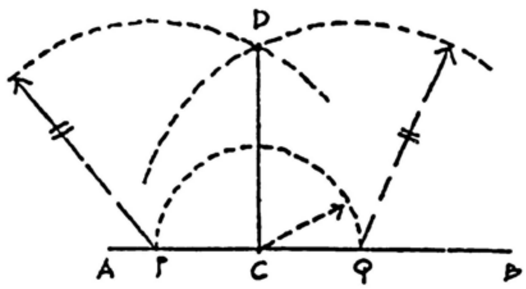
1. Given Straight line AB, cut line AB at point P.
2. With point P as centre, PB as radius, swing arc.
3. With point B as centre, BP as radius, swing arc.
4. The arcs around points P and B respectively, will intersect at Q, forming equilateral triangle PQB.
5. From point B as centre, BQ as radius, swing arc.
6. From point Q as centre, BQ as radius, swing arc.
7. The arcs around points B and Q will intersect at point R.
8. From point Q as centre, radius QR, swing arc.
9. From point R as centre, radius RQ, swing arc.
10. The arcs above points Q and R, respectively, will intersect at point C.
11. Connect point C to point B and line CB will be perpendicular to line AB.





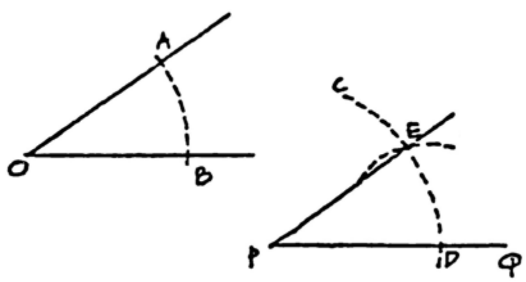
KEYWORDS
Bisector, angle, line, perpendicular, straight line

# Perpendiculars and Bisectors
# 2.1



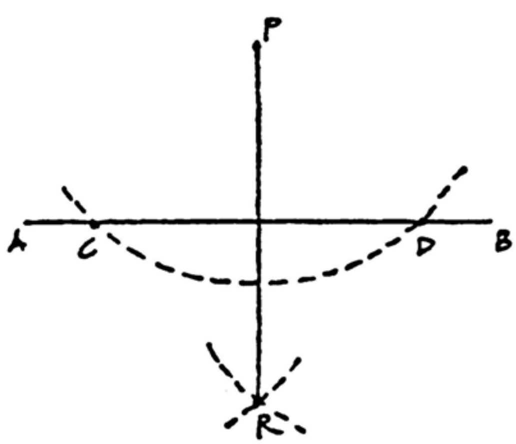

**2.1.4 To draw a straight line perpendicular to a given straight line from any point on it.**

1. Given a straight line AB and point C on it.
2. With point C as centre, set compass to any radius (in this case CQ) and draw a semi-circle around the centre.
3. With point P as centre, draw arc radius equal to PQ.
4. With point Q as centre, draw arc radius equal to QP.
5. The arcs about point P and Q will intersect at point D.
6. Connect point C to point D and the line CD will be the desired perpendicular.



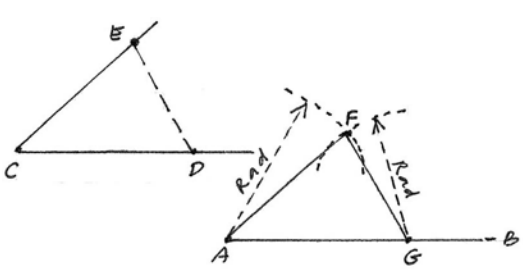

**2.1.5 To construct an angle equal to a given angle.**

1. Given angle AOB, with point O as centre, draw arc with radius OB to cut angle at points A, B.
2. Placing compass at point P, on line PQ, keeping same radius (OB), draw arc CD, where radius PD = radius OB.
3. Set compass to equal distance from point A and point B on angle AOB (radius = AB).
4. With radius AB, set compass on point D and swing arc to intersect arc around point P at point E.
5. Connect point E to point P on line PQ.  EPQ is the desired angle.




**2.1.6 To draw a straight line perpendicular to a given straight line from a given point without.**

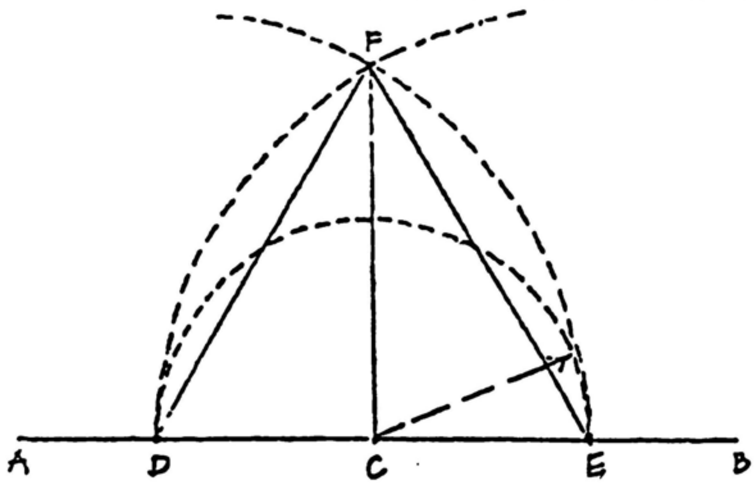
1. Given line AB and point P.
2. From point P, draw arc to cut line AB at points C and D respectively.
3. Bisect line CD, locating point R by doing so.
4. Connect point P and point R.  Line PR is the required perpendicular.




**2.1.7 To construct an angle equal to a given angle (Euclid I.23) at a given point.**

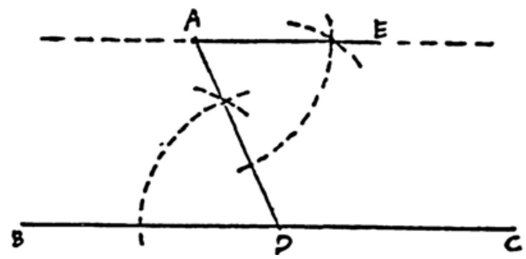
1. Given angle ECD and point A.
2. On line AB, layout length equal to CD, locating point G.
3. Connect point E to point D, constructing line ED.
4. Using point A as centre, radius equal to line CE, swing an arc.
5. Using point G as centre, radius equal to line CD, swing an arc to intersect the first arc at point F.
6. Connect point F to point A.  EAF is the required angle equal to angle ECD.

KEYWORDS
Bisector, angle, line, perpendicular, straight line



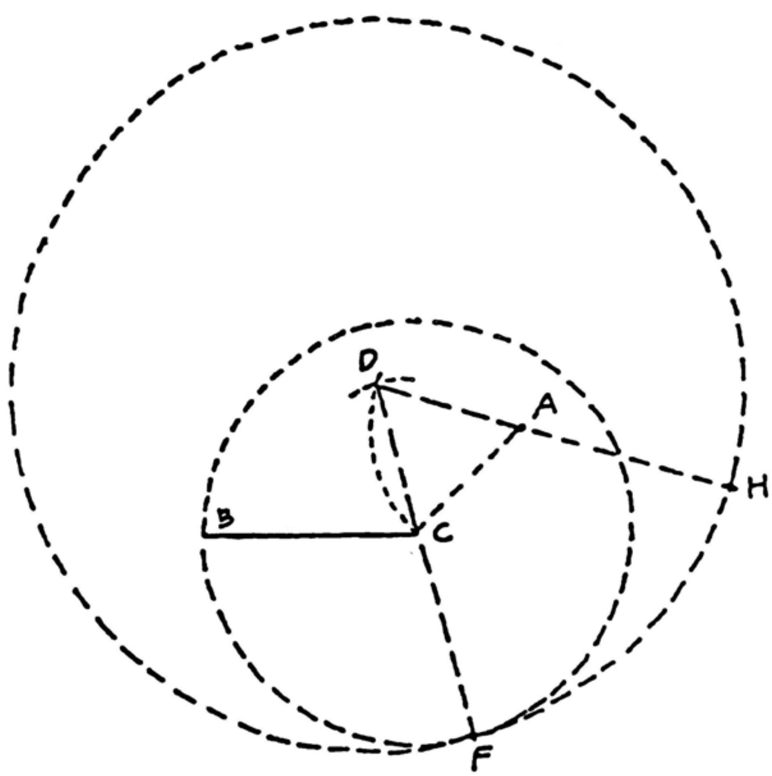

**2.2.1 To draw a straight line perpendicular to a given straight line (Euclid I.11).**

1. On a given straight line ACB, set compass point at C, draw arc DE about C.
2. From point E, draw arc, radius ED.
3. From point D, draw arc, radius DE to meet first arc at point F.
4. DEF is an equilateral triangle.




**2.2.2 To draw a straight line parallel to a given straight line through a given point (Euclid I.31).**

1. Connect given point A to given straight line BC at point D, using any angle ($45^o$ or $60^o$), constructing angle ADB.
2. Construct angle EAD equal to angle ADB.
3. The straight line EA is parallel to the given straight line BDC.



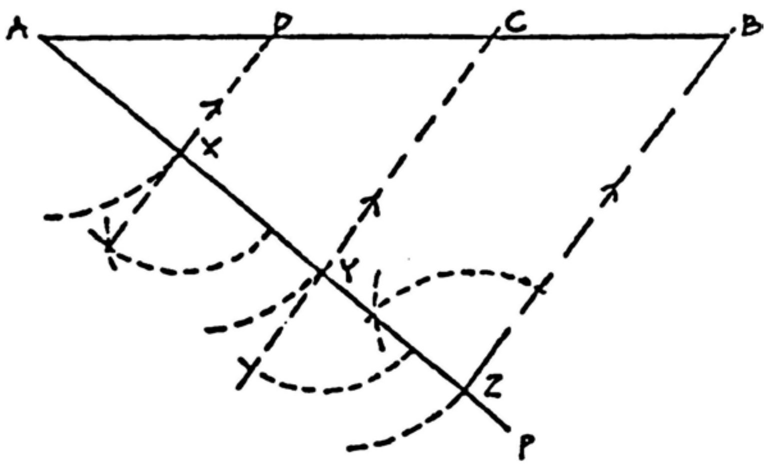

**2.2.3: To draw a straight line equal in length to a given straight line from a given point (Euclid I.2)**

1. Join point C on a given straight line BC to given point A.
2. On AC, construct equilateral triangle ACD.
3. With C as centre, radius CB describes a circle.
4. Produce line DC to meet the circumference to the circle about C at F.
5. With D as centre, radius DF, describe a circle.
6. Produce line DA to meet the circumference of the circle about D at H.
7. The line AH is equal to the given line BC.

In a large scale layout (i.e. a garden), it can often be difficult to replicate a given length, especially if natural obstacles such as trees or rocks interfere. By using this construction will allow that difficulty to be overcome by using pegs and line.

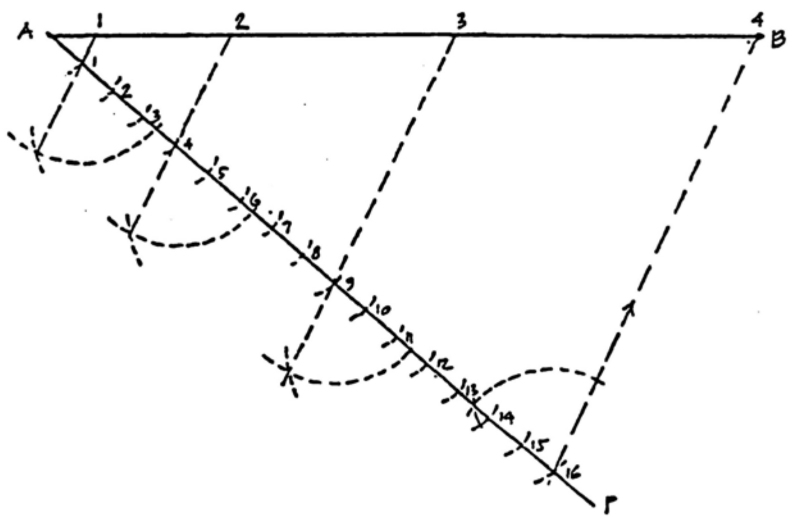
KEYWORDS
Equilateral triangle, straight line, parallel lines

# To Divide a Line
# 2.3



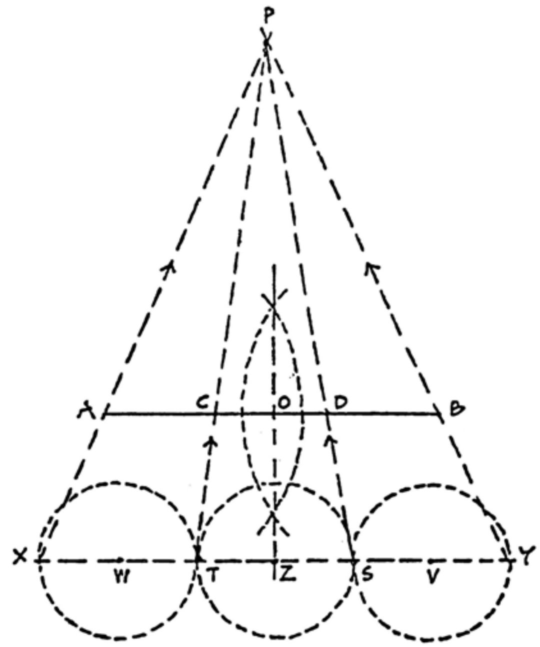

**2.3.1 To divide a given straight line into any number of equal parts - method 1.**

1. Given straight line AB, draw line AP at any angle to AB.
2. On line AP, mark off suitable length (ex. AX, XY, YZ) which are equal.
3. Connect point Z to point B.
4. Draw lines from points X and Y, respectively, parallel to line ZB.
5. Given straight line AB is now divided into equal parts.




**2.3.3 To divide a given straight line into proportional parts (in this case, divide line AB into parts proportional to XZ, where X = 1,2,3,4).**

1. As in figure 1 above, draw line AP at angle to given straight ling AB.
2. Divide line AP into units of equal length, the total of which is the highest power, or the sum of the proportional parts (in this case 1, 2, 3, 4, where the first part is x = 1, $x^2 = 1$, the second part x = 2, $x^2 = 4$ and so on, to a total of 16 parts).
3. Proceed using method 1 above to divide the line.

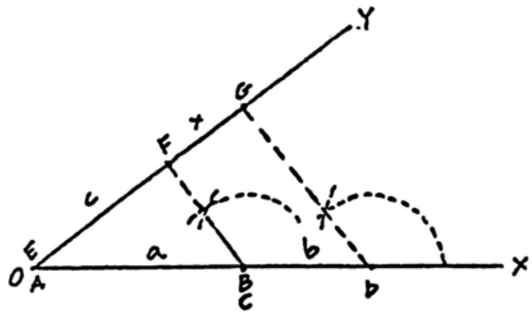



**2.3.2 Method 2 (Above)**

1. Bisect given straight line AB at O.
2. Produce bisector to point Z.
3. Through point Z, draw a circle using any radius ZT.
4. Construct line TZS through Z parallel to AB.
5. Produce diameter TZS through to points W and V respectively from point Z; WT=TZ=SZ.
6. About point W, draw a circle with radius equal to WT.
7. About point V, draw a circle with radius equal to VS.
8. Produce line XZY parallel to given straight line AB and longer than given straight line AB.
9. Produce line from point X through point A to meet line produced from point Y through point B at point P.
10. Connect points T and S to point P.
11. The line AB is now divided into equal parts.

KEYWORDS
Straight line, division, proportion

# To Divide a Line Proportionally 2.4

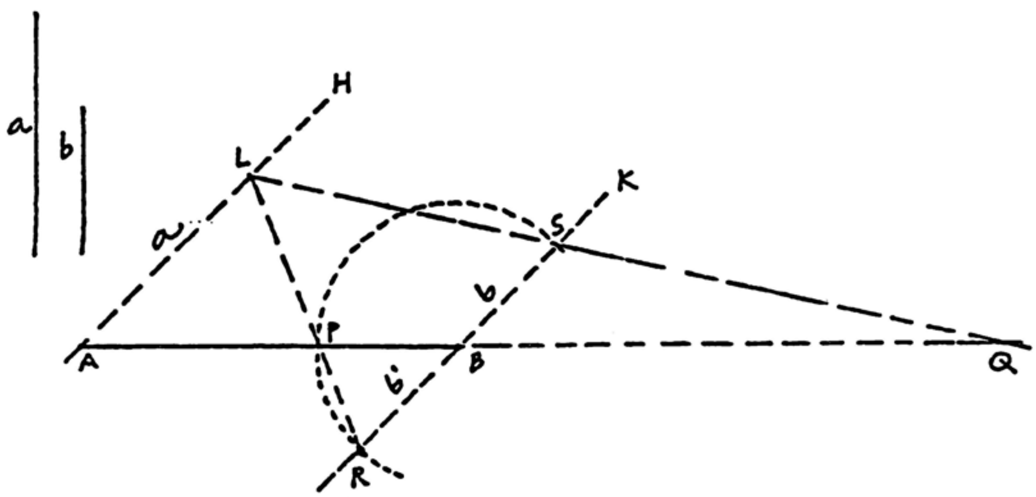



**2.4.1: To construct a fourth line proportional to three given straight lines.**

1. Given straight lines AB, CD, EF whose length measure respectively a, b, c, it is required to find a line whose length, x, is such that a/b = c/x.
2. Draw straight lines OX and OY to contain any angle constructing angle YOX.
3. On OX, lay out AB, CD.
4. On OY, lay out EF.
5. Join point B (point C) with point F.
6. From point D, draw line DG parallel to line BF.
7. FG will equal X units in length (the required fourth proportional).

Proportions are comparisons of ratios, such as a:b=c:d, or 1:2=4:8. a:b=b:c is know as a continuous proportion because b is consistent in both ratios while a:b=c:d is known as a discontinuous proportions because there is no consistent proportional. Four quantities are needed to contract a proportion. The fourth proportional is needed to develop a discontinuous proportion, which in turn allows for the development of a harmonic composition (see Section 7). An harmonic composition is one in which all the lines, elements, and sub-divisions are of the same proportion.

**2.4.2: To divide a given straight line internally and externally so that its segments may be in a given ratio (Euclid VI: Harmonic Section).**
Note: A finite straight line is said to be cut **harmonically** when it is divided internally and externally into segments which have the same ratio (in this case a and b).




1. Given line segments a and b (in ratio a:b), and straight line AB.
2. Through A and B respectively, draw any two parallel straight lines AH and BK.
3. Cut off AL equal to length a on line AH.
4. From B on BK, cut off BS equal to length b.
5. Produce the line BK through B, cutting it off at a length equal to b at R. Now BK = BS = b.
6. Join LR cutting AB at point P.
7. Join LS and produce H to intersect and cut given line AB, produced, at point Q. (AB is now cut "externally").
8. AB is now divided **harmonically** in that AP:PB = AQ:QB.

The division of a line harmonically provides the linear dimensions needed for developing an harmonic composition (see Section 7).

KEYWORDS
Parallel lines, proportion, division

*This page intentionally left blank.*